

# Dynamics of interacting phantom and quintessence dark energies

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**Abstract** We present models, in which phantom energy interacts with two different types of dark energies including variable modified Chaplygin gas (VMCG) and new modified Chaplygin gas (NMCG). We then construct potentials for these cases. It has been shown that the potential of the phantom field decreases from a higher value with the evolution of the Universe.

**Keywords** Dark energy; Chaplygin gas; quintessence; phantom energy.

## 1 Introduction

One of the outstanding developments in cosmological physics in the past decade is the discovery of the accelerated expansion of the universe, supposedly driven by some exotic dark energy (Perlmutter et al 1999; Riess et al 1998; Spergel et al 2003, 2007; Copeland et al 2006). Surprisingly, the energy density of the dark energy is two-third of the critical density ( $\Omega_\Lambda \simeq 0.7$ ) apart from dark matter ( $\Omega_m \simeq 0.3$ ). The astrophysical data shows that this sudden transition in the expansion history of the universe is marginally recent ( $z \simeq 0.7$ ) compared with the age of the universe. The nature

and composition of dark energy is still an open problem. With the thermodynamical studies of dark energy, it is conjectured that the constituents of dark energy may be massless particles (bosons or fermions) whose collective behavior resembles a kind of radiation fluid with negative pressure. Moreover, the temperature of the universe filled with dark energy will increase as the universe expands (Lima & Alacaniz 2004). The earliest proposal to explain the recent accelerated expansion was the cosmological constant  $\Lambda$  represented by the equation of state (EoS)  $p = -\rho$  (or  $w = -1$ ) having a negative pressure. In order to comply with the data, the cosmological constant has to be fine tuned up to 120 orders of magnitude (Doglov 2004), which requires extreme fine tuning of several cosmological parameters. The cosmological constant also poses a famous cosmic coincidence problem (the question of explaining why the vacuum energy came to dominate the universe very recently) (Bento et al 2002). The coincidence problem is tackled with the use of a homogeneous and time dependent scalar field  $\phi$ , in which the scalar field rolls down a potential  $V(Q)$  according to an attractor-like solution to the equations of motion (Zlatev et al 1999). But here the field has difficulties in reaching  $w < -0.7$ , while current observations favor  $w < -0.78$  with 95% confidence level (Linder 2005). Other scalar field models of dark energy include ghost condensates (Arkani-Hamed et al 2004), tachyon (Sen 2002; Setare 2007), holographic dark energy (Jamil et al 2009) and quintom (Zhang 2005). It is shown that a quintessence scalar field coupled with either a dissipative matter field, a Chaplygin gas (CG) or a tachyonic fluid solves the coincidence problem (Chimento & Jakubi 2003). These problems are alternatively discussed using anthropic principles as well (Weinberg 1987). Several other models have been proposed to explain the cosmic accelerated expansion by introducing decaying vacuum energy (Freese et al 1987; Frieman et al 1995), a

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cardassian term in the Friedmann-Robertson-Walker (FRW) equations (Freese & Lewis 2002), a generalized Chaplygin gas (GCG) (Bento et al 2006; Setare 2006, 2007) and a phantom energy ( $w < -1$ ) arising from the violation of energy conditions (Caldwell et al 2003; Babichev et al 2004; Nesseris & Perivolaropoulos 2004; Setare 2007). Another possibility is the ‘geometric dark energy’ based on the Ricci scalar  $R$  represented by  $\mathfrak{R} = R/12H^2$ , where  $H$  is the Hubble parameter (Linder 2005). Notice that  $\mathfrak{R} > 1/2$  represents accelerated expansion, and  $\mathfrak{R} > 1$  gives a super-accelerated expansion of the universe, whereas presently  $\mathfrak{R} = 1/2$ .

Models based on dark energy interacting with dark matter have been widely investigated (Setare & Vagenas 2007; Sami et al 2005; Li et al 2008; Wu & Yu 2007; Wang et al 2007; Jamil & Rashid 2008, 2009; Zimdahl & Pavon 2007; Setare 2007; Curbelo et al 2006; Mota 2004). These models yield stable scaling solution of the FRW equations at late times of the evolving universe. Moreover, the interacting CG allows the universe to cross the phantom divide (the transition from  $w > -1$  to  $w < -1$ ), which is not permissible in pure CG models. In fact it is pointed out that a phantom divide (or crossing) is possible only if the cosmic fluids have some interaction (Vikman 2005). It is possible that this interaction can arise from the time variation of the mass of dark matter particles (Zhang et al 2006). It is shown that the cosmic coincidence problem is fairly alleviated in the interacting CG models (Campo et al 2006). This result has been endorsed with interacting dark energy in (Sadjadi & Alimohammadi 2006). There is a report that this interaction is physically observed in the Abell cluster A586, which in fact supports the GCG cosmological model and apparently rules out the  $\Lambda$ CDM model (Bertolami et al 2007). However, a different investigation of the observational  $H(z)$  data rules out the occurrence of any such interaction and favors the possibility of either more exotic couplings or no interaction at all (Wei & Zhang 2007; Umar et al 2010, 2009). The consideration of interaction between quintessence and phantom dark energies can be motivated from the quintom models (Zhang 2005). In this context, we have investigated the interaction of the dark energy with dark matter by using a more general interaction term. We have focused on the inhomogeneous EoS for dark energy as these are phenomenologically relevant.

The outline of the paper is as follows. In the section II, we present a general interacting model for our dynamical system. Following (Chattopadhyay & Debnath 2010), we consider the two interacting dark energy

models like variable modified Chaplygin gas (VMCG) and new modified Chaplygin gas (NMCG) interact with phantom field in sections III and IV. We found the phantom potential in these scenarios. Finally, we present our conclusion.

## 2 The model

We assume the background to be a spatially flat isotropic and homogeneous FRW spacetime, given by

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (1)$$

where  $a(t)$  is the scale factor. The corresponding Einstein field equations are

$$3H^2 = \rho_{\text{tot}} \quad (2)$$

and

$$6(\dot{H} + H^2) = -(\rho_{\text{tot}} + 3p_{\text{tot}}). \quad (3)$$

Here  $\rho_{\text{tot}}$  and  $p_{\text{tot}}$  represent the total energy density and isotropic pressure respectively ( $8\pi G = c = 1$ ). Moreover, the energy conservation for our gravitational system is given by

$$\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0. \quad (4)$$

Suppose we have a two-component model of the form

$$\rho_{\text{tot}} = \rho_1 + \rho_2, \quad (5)$$

and

$$p_{\text{tot}} = p_1 + p_2. \quad (6)$$

Here  $\rho_1$  and  $p_1$  denote the energy density and pressure of quintessence and  $\rho_2$ ,  $p_2$  denote the energy density and pressure of phantom dark energy. The stress energy tensor for matter-energy is

$$T_{\mu\nu} = -\partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left[\frac{\sigma}{2}g^{\beta\delta}\partial_\beta\partial_\delta\Phi + V(\Phi)\right]. \quad (7)$$

By assuming that the phantom field is evolving in an isotropic homogenous universe and that  $\Phi$  is merely function of time, from Eq. (7) one can extract energy density and pressure as

$$\rho_1 = \frac{\sigma}{2}\dot{\Phi}^2 + V(\Phi), \quad (8)$$

$$p_1 = \frac{\sigma}{2}\dot{\Phi}^2 - V(\Phi). \quad (9)$$

Here  $\sigma = -1$  corresponds to the phantom field while  $\sigma = +1$  represents the standard scalar field which represents the quintessence field, also  $V(\Phi)$  is the potential.

In this case, the equation of state  $w$  is given by

$$w = \frac{p_1}{\rho_1} = \frac{\sigma\dot{\Phi}^2 - 2V(\Phi)}{\sigma\dot{\Phi}^2 + 2V(\Phi)}. \quad (10)$$

We observe that it results in the violation of the null energy condition  $\rho_1 + p_1 = \sigma\dot{\Phi}^2 > 0$ , if  $\sigma = -1$ . Since the null energy condition is the basic condition, its violation yields other standard energy conditions to be violated likewise dominant energy condition ( $\rho_1 > 0$ ,  $\rho_1 \geq |p_1|$ ) and the strong energy condition ( $\rho_1 + p_1 > 0$ ,  $\rho_1 + 3p_1 > 0$ ). Due to the energy condition violations, it makes the failure of cosmic censorship conjecture and theorems related to black hole thermodynamics. The prime motivation to introduce this weird concept in cosmology does not come from the theory but from the observational data. According to the forms of dark energy density and pressure (8) and (9), one can easily obtain the kinetic energy and the scalar potential terms as

$$\dot{\Phi}^2 = \frac{1}{\sigma}(1 + \omega)\rho_1, \quad (11)$$

$$V(\Phi) = \frac{1}{2}(1 - \omega)\rho_1. \quad (12)$$

### 3 Variable Modified Chaplygin Gas

Firstly, let us suppose that we have variable modified Chaplygin gas (VMCG) representing the dark energy and is given by (Debnath 2007)

$$p_2 = A_1\rho_2 - \frac{B_0a(t)^{-n}}{\rho_2^\alpha}, \quad (13)$$

where  $0 \leq \alpha \leq 1$ ,  $0 \leq A_1 \leq 1$ ,  $B_0$  and  $n$  are constant parameters. The Chaplygin gas behaves like dust in the early evolution of the universe and subsequently grows to an asymptotic cosmological constant at late time when the universe is sufficiently large. In the cosmological context, the Chaplygin gas was first suggested as an alternative to quintessence (Curbelo et al 2006). Later on, the Chaplygin gas state equation was extended to a modified form by adding a barotropic term (Benaoum 2002; Debnath et al 2004; Jamil et al 2009). Recent supernovae data also favor the two-fluid cosmological model with Chaplygin gas and matter (Panotopoulos 2008). Suppose that the phantom field interacts with (VMCG), so under this interaction (supposing the interaction term is  $Q$ ) the continuity equations can be written as

$$\dot{\rho}_1 + 3H(\rho_1 + p_1) = Q, \quad (14)$$

$$\dot{\rho}_2 + 3H(\rho_2 + p_2) = -Q. \quad (15)$$

In case of  $Q = 0$ , we arrive at the non-interacting situation while  $Q > 0$  exhibit a transfer of energy from the one fluid of density  $\rho_1$  to other fluid of density  $\rho_2$ . In order to solve the above continuity equations different forms of  $Q$  have been considered. Here we will proceed to solve the continuity equation (15) by taking  $Q = 3\delta H\rho_2$  ( $\delta$  is a coupling constant), so we get

$$\rho_2 = \left[ \frac{3B_0(1 + \alpha)}{[3A_1(1 + \alpha) + 3(1 + \alpha)(1 + \delta) - n]a^n} + \frac{C}{a^{3(1 + \alpha)(1 + \delta + A_1)}} \right]^{\frac{1}{1 + \alpha}}, \quad (16)$$

where  $C$  is the constant of integration. One can be seen that if  $n = 0$  and  $A, B$  approach to zero, then  $\rho_2 \sim a^{-3(1 + \delta)}$ . Now for simplicity, we choose  $V = m\dot{\Phi}^2$ , where  $m$  is a positive constant. So using (14) and (16), we obtain the kinetic term as

$$\begin{aligned} \dot{\Phi}^2 &= C_1 a^{-\frac{6\sigma}{\sigma + 2m}} + \frac{6\sigma(1 + \alpha)}{(-2mn + (6 - n + 6\alpha)\sigma)} \\ &\times \left( \frac{3B_0(1 + \alpha)}{-n + 3(1 + \alpha)(1 + \delta + A_1)} \right)^{\frac{1}{1 + \alpha}} a^{-\frac{n}{1 + \alpha}} \\ &\times {}_2F_1 \left[ x, -\frac{1}{1 + \alpha}, 1 + x, -Y a^{n - 3(1 + \alpha)(1 + \delta + A_1)} \right], \end{aligned} \quad (17)$$

and the potential energy has the form

$$\begin{aligned} V &= mC_1 a^{-\frac{6\sigma}{\sigma + 2m}} + \frac{6m\sigma(1 + \alpha)}{(-2mn + (6 - n + 6\alpha)\sigma)} \\ &\times \left( \frac{3B_0(1 + \alpha)}{-n + 3(1 + \alpha)(1 + \delta + A_1)} \right)^{\frac{1}{1 + \alpha}} a^{-\frac{n}{1 + \alpha}} \\ &\times {}_2F_1 \left[ x, -\frac{1}{1 + \alpha}, 1 + x, -Y a^{n - 3(1 + \alpha)(1 + \delta + A_1)} \right], \end{aligned} \quad (18)$$

where

$$\begin{aligned} Y &= \frac{C(-n + 3(1 + \alpha)(1 + \delta + A_1))}{3B_0(1 + \alpha)}, \\ x &= \frac{2mn + (n - 6(1 + \alpha))\sigma}{(1 + \alpha)(2m + \sigma)(-n + 3(1 + \alpha)(1 + \delta + A_1))}, \end{aligned}$$

and  $C_1$  is the constant of integration. From the expression (18) it is clear that the potential energy is a function of scale factor  $a$ . The graphs represented by Fig. 1 and Fig. 2 show that  $\phi$  increases with the passage of time while the  $V$  decreases with the increase of cosmic time  $t$ .

### 4 New Modified Chaplygin Gas

The model which behaves as a dark matter (radiation) at the early stage and  $X$ -type dark energy at

late stage is the New Modified Chaplygin Gas (NMCG) (Zhang et al 2006; Chattopadhyay & Debnath 2008)

$$p_2 = \beta \rho_2 + \frac{w A_2 a^{-3(1+w)(1+\alpha)}}{\rho_2^\alpha}, \quad A_2 > 0, \quad \beta > 0. \quad (19)$$

In view of (second energy eq.), the energy density of the (NMCG) can be expressed as

$$\rho_2 = \left[ \frac{A_2 w a^{-3(1+w)(1+\alpha)}}{w - \delta - \beta} + C_1 a^{-3(1+\delta+\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}} \quad (20)$$

Now for simplicity, we again choose  $V = m\dot{\Phi}^2$ , where  $m$  is a positive constant. So using (14) and (20), we obtain the kinetic term as

$$\begin{aligned} \dot{\Phi}^2 &= C_2 a^{-\frac{6\sigma}{\sigma+2m}} - \frac{2a^{-3(1+w)\sigma}}{(2m(1+w) + \sigma(-1+w))} \\ &\quad \times {}_2F_1 \left[ x_1, -\frac{1}{1+\alpha}, 1+x_1, Y_1 a^{3(1+\alpha)(w-\beta-\delta)} \right], \end{aligned} \quad (21)$$

and the potential energy has the form

$$\begin{aligned} V &= m C_2 a^{-\frac{6\sigma}{\sigma+2m}} - \frac{2ma^{-3(1+w)\sigma}}{(2m(1+w) + \sigma(-1+w))} \\ &\quad \times {}_2F_1 \left[ x_1, -\frac{1}{1+\alpha}, 1+x_1, Y_1 a^{3(1+\alpha)(w-\beta-\delta)} \right], \end{aligned} \quad (22)$$

where

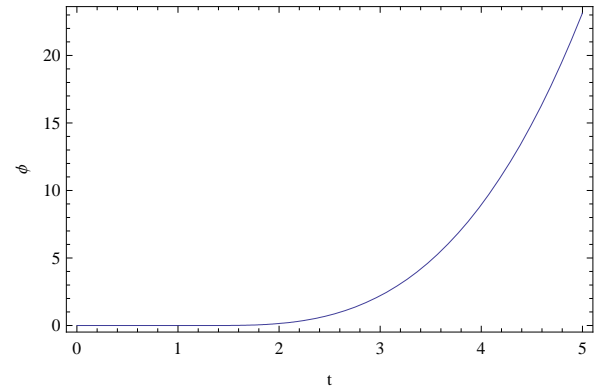
$$\begin{aligned} Y_1 &= \frac{C_1(-w + \beta + \delta)}{A_2 w 2}, \\ x_1 &= \frac{-2m(1+w) + \sigma(1-w)}{(1+\alpha)(2m+\sigma)(w-\beta-\delta)}, \end{aligned}$$

and  $C_2$  is the constant of integration.

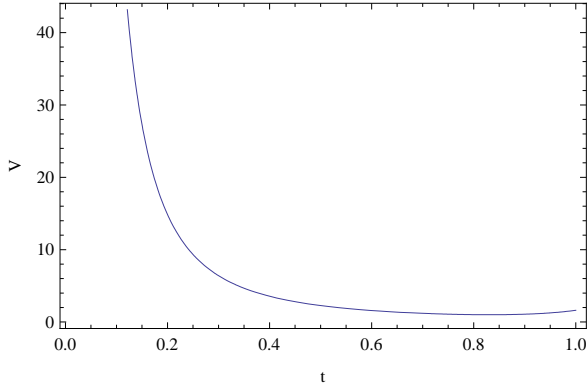
## 5 Discussion

In this work, we have considered the interacting scenario of the universe, in which phantom energy interacts with two different types of dark energies including variable modified Chaplygin gas (VMCG), new modified Chaplygin gas (NMCG). By considering some particular form of interaction term, we have constructed the potential of the phantom field. By looking at the energy conservation equations (14) and (15) it is found that the energies of the (VMCG) and (NMCG) are getting transferred to the phantom field. With the help of graphs we studied the variations of  $V$  and  $\phi$  with the variation of the cosmic time. From the figures we

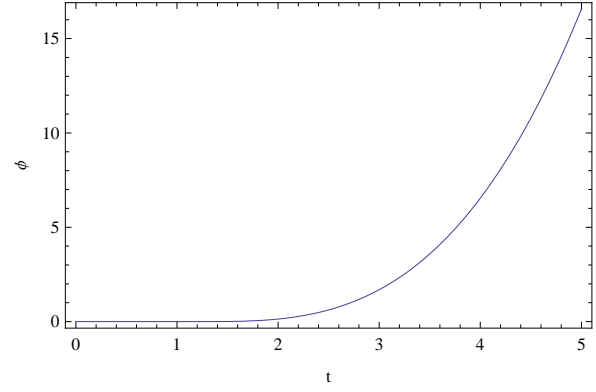
see that the potential decreases from the lower value with the evolution of the universe. Thus in the presence of an interaction, the potential decreases and the field decreases with the evolution of the universe.



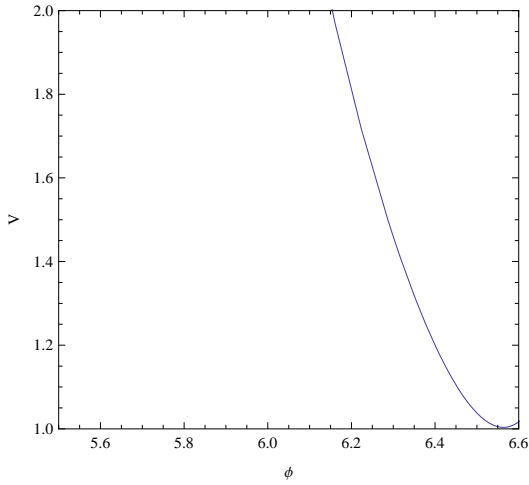
**Fig. 1** The variation of  $\Phi$  against cosmic time  $t$  with particular values of parameters  $A_1 = 0.1, B_0 = 0.1, \delta = 0.005, \alpha = 0.5, C = 1, C_1 = 1, n = 0.9, m = 0.7$ .



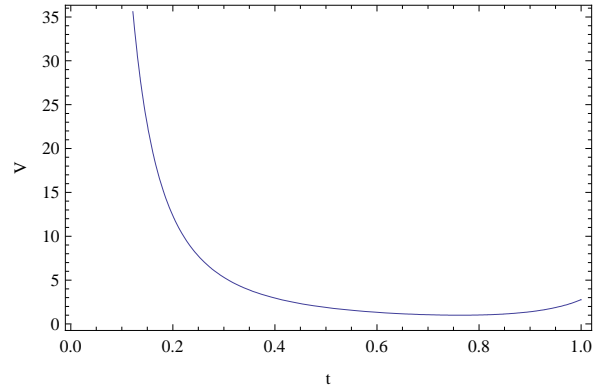
**Fig. 2** The variation of  $V$  against cosmic time  $t$   $\sigma = -1$  (phantom field) in VMCG with particular values of parameters  $A_1 = 0.1, B_0 = 0.1, \delta = 0.005, \alpha = 0.5, C = 1, C_1 = 1, n = 0.9, m = 0.7$ .



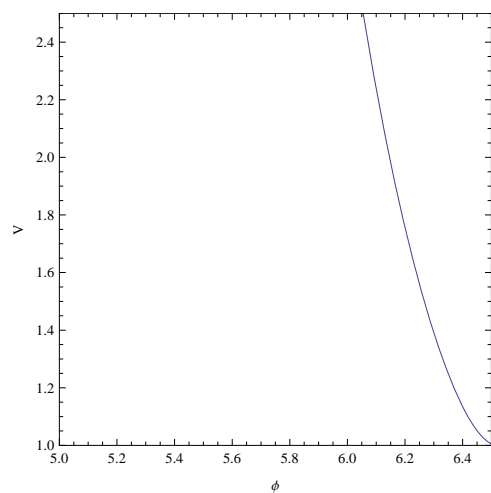
**Fig. 4** The variations of  $\Phi$  and  $t$  against cosmic time  $t$  respectively and Fig. 6 represents the variation of  $V$  against  $\Phi$  for  $\sigma = -1$  (phantom field) in NMCG with particular values of parameters  $A_2 = 0.1, \beta = 0.2, \delta = 0.005, \alpha = 0.5, C_1 = 1, C_2 = 1, w = -0.9, m = 0.7$ .



**Fig. 3** The variation of  $V$  against  $\Phi$  for  $\sigma = -1$  (phantom field) in VMCG with particular values of parameters  $A_1 = 0.1, B_0 = 0.1, \delta = 0.005, \alpha = 0.5, C = 1, C_1 = 1, n = 0.9, m = 0.7$ .



**Fig. 5** The variation of  $V$  against  $t$  for  $\sigma = -1$  (phantom field) in NMCG with particular values of parameters  $A_2 = 0.1, \beta = 0.2, \delta = 0.005, \alpha = 0.5, C_1 = 1, C_2 = 1, w = -0.9, m = 0.7$ .



**Fig. 6** The variation of  $V$  against  $\Phi$  for  $\sigma = -1$  (phantom field) in NMCG with particular values of parameters  $A_2 = 0.1, \beta = 0.2, \delta = 0.005, \alpha = 0.5, C_1 = 1, C_2 = 1, w = -0.9, m = 0.7$ .

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